

Cover Sheet

Title: **Experimental Realization of Controlling Chaos in the Periodically Switched Nonlinear Circuit**

Authors: Takuji KOUSAKA†, Yoshito YASUHARA†,
 Tetsushi UETA‡, and Hiroshi KAWAKAMI §

Facilities: †Dept. of Electronic and Electrical Engineering,
 Faculty of Engineering, Fukuyama University
 ‡Dept. of Information Science and Intelligent Systems,
 Faculty of Engineering, The University of Tokushima
 §Dept. of Electrical and Electronic Engineering,
 Faculty of Engineering, The University of Tokushima

Corresponding Author:

Takuji Kousaka, Lecturer
Dept. of Electronic and Electrical Engineering,
Faculty of Engineering, Fukuyama University
1, Gakuen-cho, Fukuyama, Hiroshima 729-0292, Japan

Telephone: +81-84-936-2111 ext.4152

FAX: +81-84-936-2023

E-mail: kousaka@fuee.fukuyama-u.ac.jp

Abstract This letter presents an experimental confirmation of controlling the chaotic behavior to a target unstable periodic orbit when the periodically switched nonlinear circuit has a chaotic attractor. The pole assignment for the corresponding discrete system derived from such a non-autonomous system via Poincaré mapping works effectively, and the control unit is easily realized by the window comparator, sample-hold circuits, and so on.

Index terms: Chaos, control of chaos, periodic switch, nonlinear circuit.

1 Introduction

Controlling chaos might be one of the effective engineering application of chaos (as surveyed in [Chen, 2000]). In the piecewise linear systems, there have been numerous theoretical and experimental investigations on the controlling chaos [Poddar et al., 1995.1] [Poddar et al., 1995.2, Poddar et al., 1998, Bernardo & Chen, 2000, Bueno & Marrero, 2000, Saito & Mitsubori, 1995, Tsubone & Mitsubori, 1998, Kousaka et al., 2001.1]. For example, [Poddar et al., 1995.1, Poddar et al., 1995.2, Poddar et al., 1998, Bernardo & Chen, 2000] [Bueno & Marrero, 2000] showed theoretical and experimental implementation for DC-DC power converters. Since current or voltage controlled converters have wide industrial applications, controlling chaos for such system is important from a practical point of view. On the other hand, a controlled circuit which has been demonstrated in [Saito & Mitsubori, 1995, Tsubone & Mitsubori, 1998] is a 2-dimensional circuit with piecewise linear hysteresis characteristics. We have lately developed a simple hybrid system exhibiting chaos and realized its chaos control [Kousaka et al., 2001.1]. This fact directly indicates that one-dimensional return map is derived rigorously by using the exact solution of the circuit equation. By adding a controller manipulating the slope around an unstable periodic point, controlling chaos is almost achieved. These methods take positive advantage of the controlling chaos in the low dimensional piecewise linear systems. However, if the system has a nonlinear term with non-smooth characteristics or the dynamics of the system is described by high dimensional piecewise linear system, there are no methods for controlling chaos in such systems. In order to control these systems, we proposed a control method of the chaos in the periodically switched nonlinear systems and controlled the unstable periodic orbit in the numerical simulation [Kousaka et al., 2001.2]. This method can be used as a general technique for controlling chaos, however, the control function was not demonstrated in the laboratory experiments. This letter presents the first report of the experimental control of the chaotic attractor in the periodically switched nonlinear circuit. The controller is realizable because all information about the circuit can be obtained from the composite Poincaré map, and the control unit is easily realized by the window comparator, sample-hold circuits, and so on.

2 Circuit dynamics and switching action

Consider the experimental control of the Rayleigh type oscillator containing a periodic switch shown in Fig.1. We assume that the nonlinear characteristics of the resistor is a cubic function

as $G(v) = -a_1v + a_3v^3$. Figure 2 shows the behavior of the trajectory. An external periodic signal of the positions a and b are alternatively closed. τ is the switching period and $0 \leq \theta \leq 1$ is the duty ratio of the switching, i.e., the period while the switch is turned toward a is expressed by $\theta\tau$, and $0 \leq \theta \leq 1$, in contrast, the period while the switch is turned toward b is expressed by $(1 - \theta)\tau$. Note that the unstable fixed point exists locally on both sides of the switching manifold. This means that the system flow is continuous inside each region, but is discontinuous on the switching manifold. We fix the parameters as,

$$\begin{aligned} L &= 50[\text{mH}], C_1 = C_2 = 0.1[\mu\text{F}], R_0 = 0[\Omega], R_1 = 987[\Omega], R_2 = 281[\Omega], \\ r &= 70.7[\Omega], E_1 = 1.87[\text{V}], E_2 = 2.1[\text{V}], R_3 = 10.0[\text{k}\Omega], R_4 = 8.6[\text{k}\Omega], \\ R_{5a} &= 766[\Omega], R_{5b} = 4.5[\text{k}\Omega], a_1 = 2.145 \times 10^{-3}, a_3 = 6.9 \times 10^{-5}. \end{aligned} \quad (1)$$

With these parameters, the system has a chaotic attractor via period doubling cascade as shown in Fig. 3[Kousaka et al., 2001.2]. In this letter, we choose E_2 as the control parameter and consider the circuit realization of the controlling chaos.

3 Experimental Realization of Control Algorithm

We briefly explain the control method of the nonlinear system containing a periodic switch. By utilizing the periodicity of the switching action, we first construct the local mapping, and the Poincaré mapping is constructed as a composite map of local mappings. The derivative of the Poincaré map is equal to the product of the derivative of each local maps. With above parameters, the location($i^*[\text{mA}], v^*[\text{V}]$) of the unstable fixed point is $(-3.57, 0.36)$, and its multipliers are $(-0.25, -2.42)$, respectively. By using the following state feedback u_k :

$$u_k = G_1(i_k - i^*) + G_2(v_k - v^*). \quad (2)$$

the unstable 1-periodic orbit in the chaotic attractor can be stabilized due to the linear control technique with stable pole assignment. We now place the poles to realized dead-beat control. From this, we can obtain the control gain $G_1 = 495.2$ and $G_2 = 2.47$. Mathematically, more detailed discussion of the control method is given in [Kousaka et al., 2001.2].

Figure 4 shows the circuit diagram of the controller. The control unit is easily realized by the window comparator, sample-hold circuits, and so on. When the following condition is satisfied, the control is started.

$$|i_k - i^*| < \delta_1, \quad |v_k - v^*| < \delta_2, \quad (3)$$

where we fix the δ -neighborhood of the target fixed point, $\delta_1 = 0.1[\text{mA}]$ and $\delta_2=0.1[\text{V}]$. That is to say, when the orbit reaches δ -neighborhood, then it is stabilized by applying the control parameter perturbation described above.

4 Experimental Results and Conclusions

Figure 5 shows the laboratory simulation of the stabilized 1-periodic orbit. The impulsive wave in Fig. 5(a) and (b) denote the period of the switching period τ . Figure 6 shows the transition from the chaotic attractor to the 1-periodic orbit. From the experimental results, We find out the robust about 5 percent perturbation in the input voltage E_2 , too.

The method proposed by [Kousaka et al., 2001.2] is a general technique of controlling chaos in the interrupted electric circuit with a periodic switch. In the near future, we will try to apply the developed method to higher-dimensional circuitry and the system which has m periodic switches in the laboratory experiments.

References

- [Bernardo & Chen, 2000] Bernardo, M. di & Chen, G. [2000] "Controlling Bifurcations in Nonsmooth Dynamical Systems," *Controlling Chaos and Bifurcations in Engineering Systems*, Ed. G. Chen, (CRC Press), Chap. 18, pp. 391-416.
- [Bueno & Marrero, 2000] Bueno, R.S. & Marrero, J.L.R. Marrero. [2000] "Application of the Ogy Method to the Control of Chaotic DC-DC Converters: Theory and Experiments," *in Proc.IEEE/ISCAS*, 369–372.
- [Chen, 2000] Chen, G. [2000] "Controlling Chaos and Bifurcations in Engineering Systems," (CRC Press).
- [Kousaka et al., 2001.1] Kousaka, T. Ueta, T. & Kawakami, H. [2001] "Chaos in a simple hybrid system and its control," *Electron. Lett.*, **37**(1), 1–2.
- [Kousaka et al., 2001.2] Kousaka, T. Ueta, T. & Kawakami, H. [2001] "Chaos and Control of Periodically switched Nonlinear Systems," *Latin American Applied Research Journal*, **31**(3), 211–218.
- [Poddar et al., 1995.1] Poddar, G. Chakrabarty, K. & Banerjee, S. [1995] "Control of chaos in the boost converter," *Electron. Lett.*, **31**(11), 841–842.

- [Poddar et al., 1995.2] Poddar, G. Chakrabarty, K. & Banerjee, S. [1995] “Experimental Control of Chaotic Behavior of Buck Converter,” *IEEE Trans. Circuits and Systems*, **42**(8), 502–504.
- [Poddar et al., 1998] Poddar, G. Chakrabarty, K. & Banerjee, S. [1998] “Control of chaos in DC-DC converters,” *IEEE Trans. Circuits and Systems*, **45**(6), 672–676.
- [Saito & Mitsubori, 1995] Saito, T. & Mitsubori, K. [1995] “Control of chaos from a piecewise linear hysteresis circuit,” *IEEE Trans. Circuits and Systems*, **42**(12), 168–172.
- [Tsubone & Mitsubori, 1998] Tsubone, T. & Saito, T. [1998] “Stabilizing and destabilizing control for a piecewise-linear circuit,” *IEEE Trans. Circuits and Systems*, **45**(2), 172–177.

Figure captions

Figure 1: The Raleigh type oscillator containing a periodic switch.

Figure 2: Behavior of the trajectory.

Figure 3: Chaotic attractor.

- (a) Phase plane.
- (b) Time evolution of i .
- (c) Time evolution of v .

Figure 4: Construction of the controller.

- (a) The Rayleigh type oscillator containing a periodic switch and its Poincaré map.
- (b) The control vector G_1 and G_2 .
- (c) The window comparator and the state feedback.

Figure 5: Stabilized 1-periodic orbit.

- (a) Phase plane.
- (b) Time evolution of i .
- (c) Time evolution of v .

Figure 6: The transition from the chaotic attractor to stabilized 1-periodic orbit.

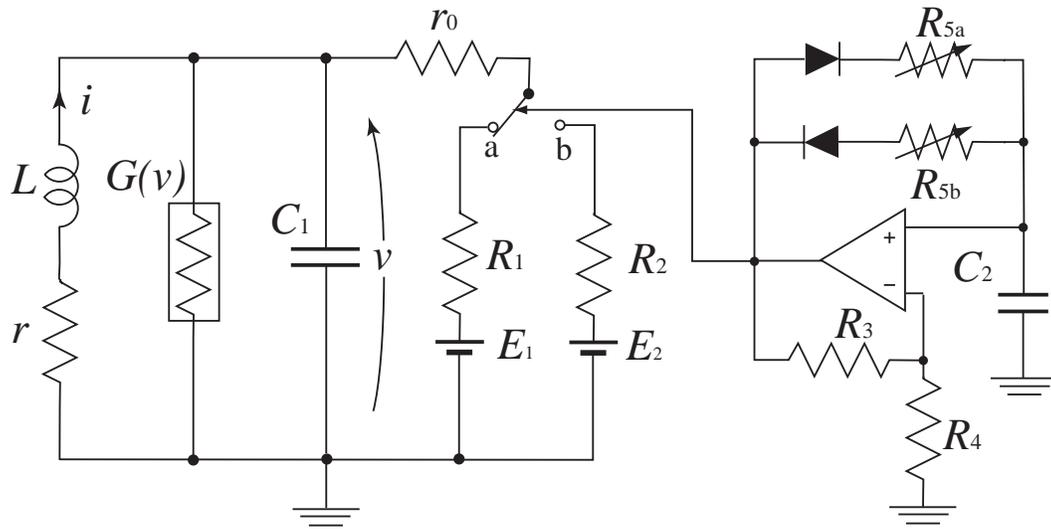


Figure 1: The Raleigh type oscillator containing a periodic switch.

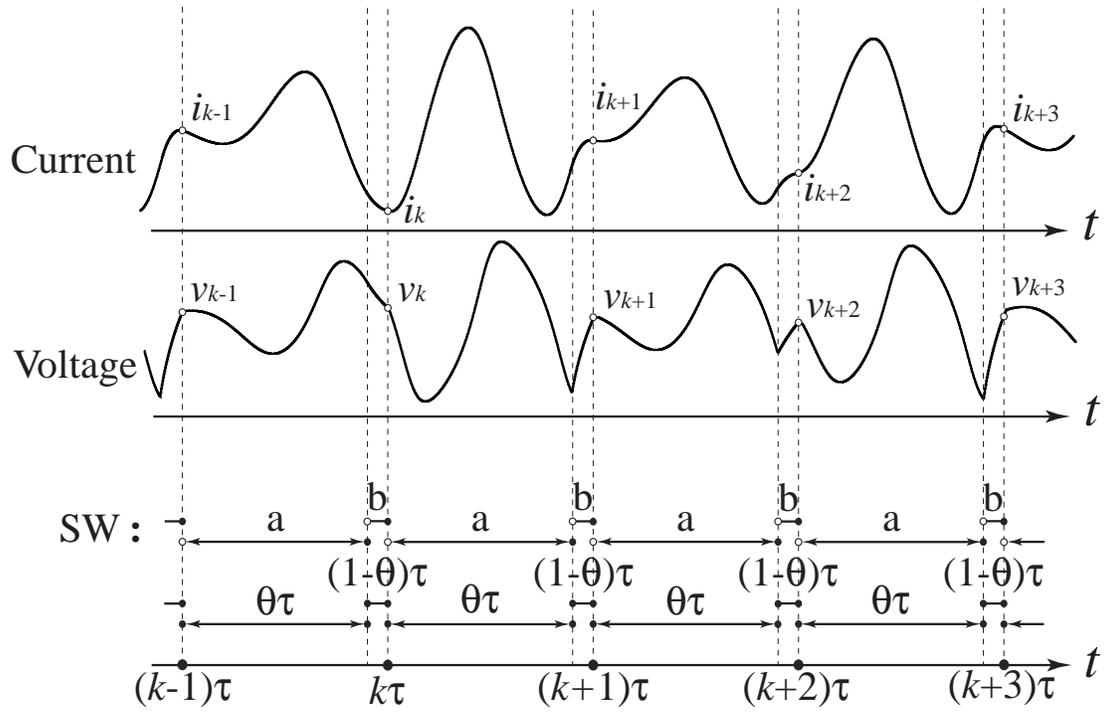
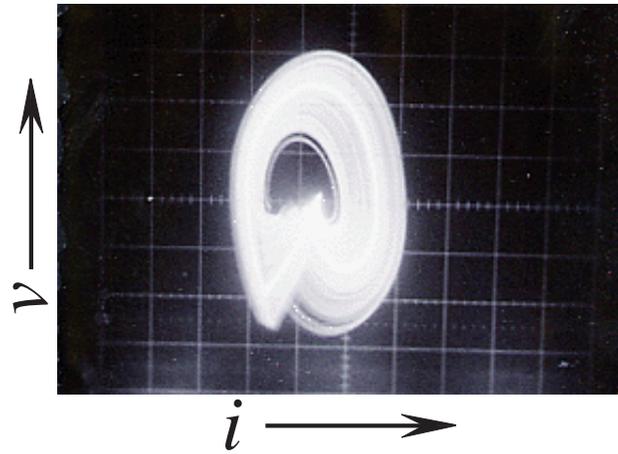
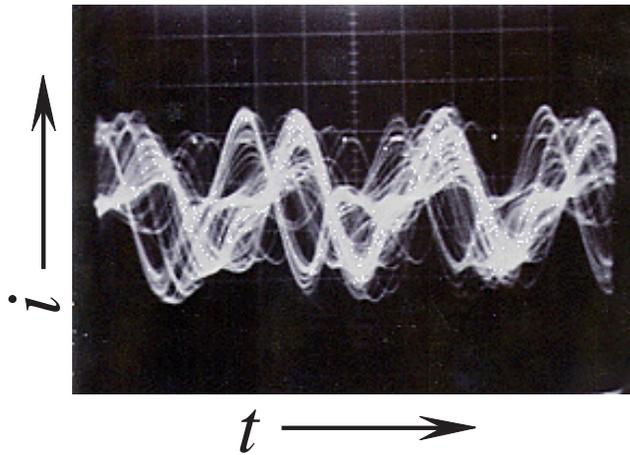


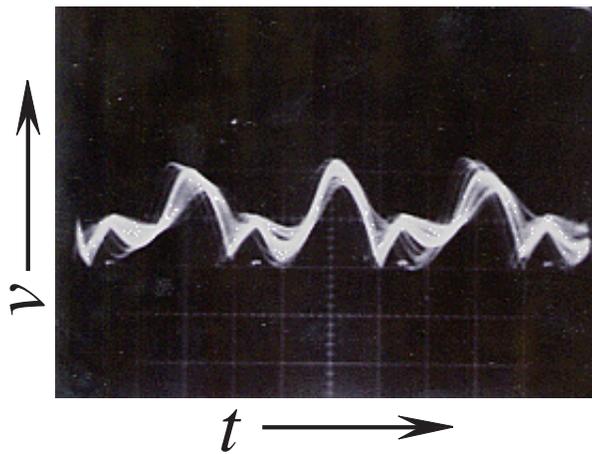
Figure 2: Behavior of the trajectory.



(a) Phase plane.

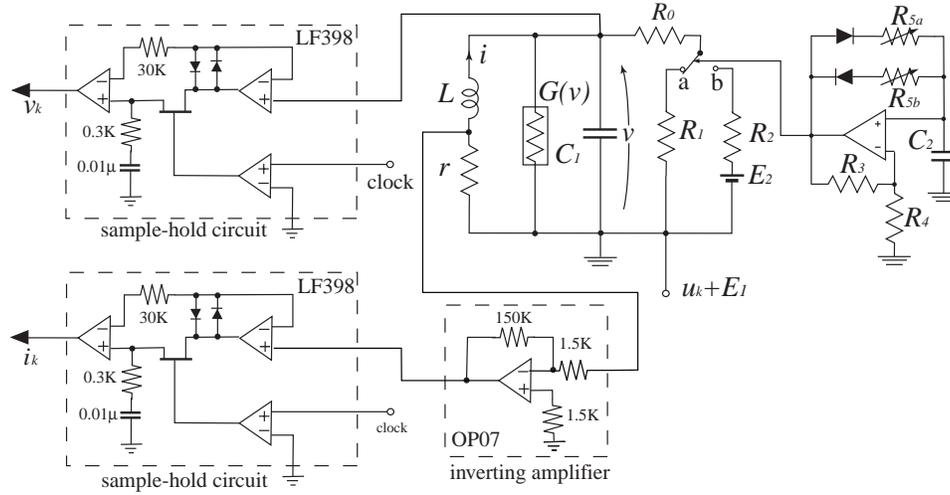


(b) Time evolution of i .

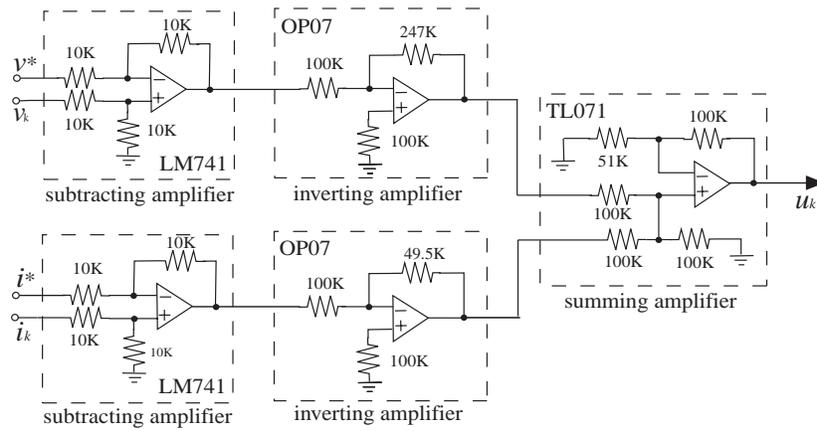


(c) Time evolution of v .

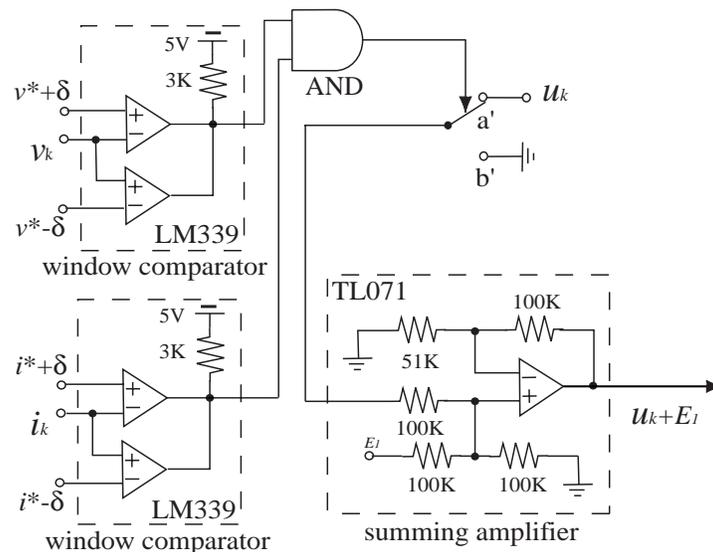
Figure 3: Chaotic attractor. ((a) i : 50[mA/div], v : 2.0[V/div], (b)(c) i : 50[mA/div], v : 5.0[V/div], t : 0.2[mS])



(a) The Rayleigh type oscillator containing a periodic switch and its Poincaré map.

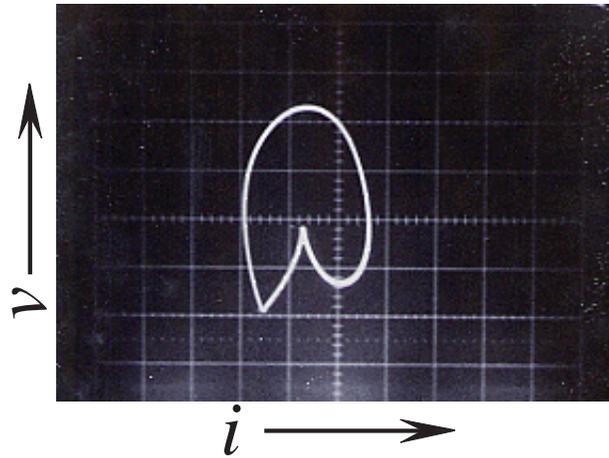


(b) The control vector G_1 and G_2 .

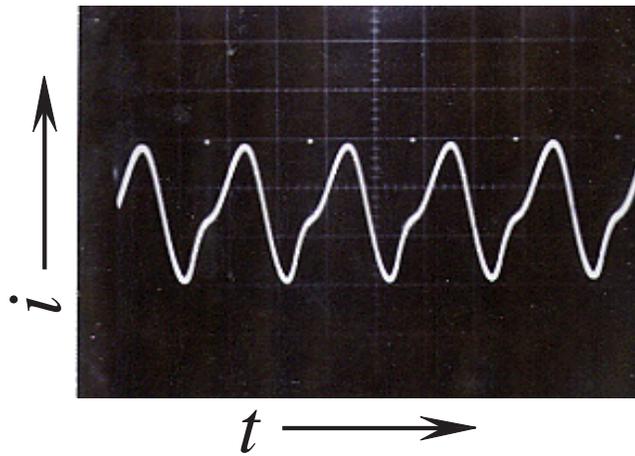


(c) The window comparator and the state feedback.

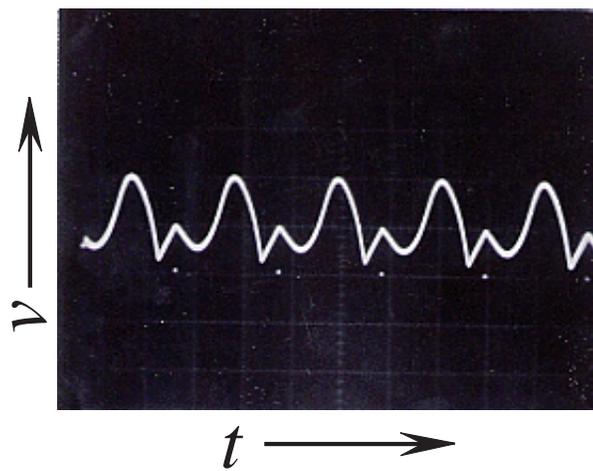
Figure 4: Construction of the controller.



(a) Phase plane.



(b) Time evolution of i .



(c) Time evolution of v .

Figure 5: Stabilized 1-periodic orbit. ((a) i : 50[mA/div], v : 2.0[V/div], (b)(c) i : 50[mA/div]), v : 5.0[V/div], t : 0.2[mS])

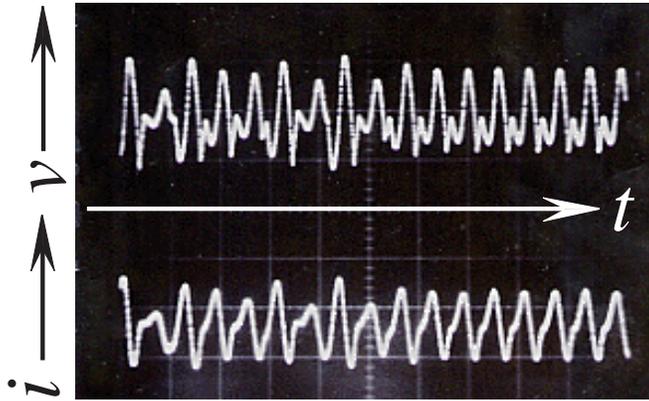


Figure 6: The transition from the chaotic attractor to stabilized 1-periodic orbit. (i : 100[mA/div], v : 5.0[V/div], t : 1.0[mS])