

Chaos Control Implementation by Interrupted Electric Circuit with Switching Delay

T. Kousaka, T. Ueta and Y. Ma

We have modeled a chaotic circuit with a switching delay and have developed a controller for the suppression of chaos. A controller that stabilizes a period-1 unstable periodic orbit in an interrupted electric circuit with a certain switching delay is also described in this paper.

Introduction: Chaos control is a method for stabilizing the unstable periodic orbits embedded in the chaotic attractor of nonlinear dynamical systems. Some methods have been proposed to control chaos in the systems described by piecewise-smooth differential equations, such as practical power converters [Bernardo & Tse, 2002, Chen, 1999]. In the case of conventional controllers, it is assumed that any switching circuit functions as designed; however, there are intrinsic delays such as computation time for determining adaptive control gain, mechanical switching time in a relay and recovery time in a diode [Banerjee et al., 2004]. These latencies are not negligible in all cases; in fact, some of them affect the control results significantly [Oda et al., 2007]. We have modeled a piecewise linear system having a switching delay. The influence of the delay is denoted by an additional interval on the map. We have developed a controller to stabilize an unstable periodic orbit embedded in the map and have confirmed its applicability by numerical simulation. In this paper, we present a chaos

circuit, including a delay element, and a controller developed by using simple elements. We also describe the laboratory experiments that yielded successful control results.

Simple switching circuits containing a delay: Figure 1 shows a circuit representation of the chaotic system including a delay element. Let T be the period of the clock and v_k be the capacitor voltage at time $t = kT$. The operation of the circuit is shown in Fig. 2. The switch comprises a flip-flop whose inputs are connected to the output of the comparator and the clock pulse from the generator. If the switch is connected to terminal A, the capacitor is charged and v_k increases. When v_k reaches v_r , the switch is connected to terminal B and then v_k decreases as the capacitor discharges. When a reset pulse arrives after every period of duration T , the switch returns to A when $v_k < v_r$. All monostable multivibrators provide the delay time τ_d ; when voltage v_k exceed v_r , the switch is connected to terminal B after τ_d seconds. This delay can simulate the latency of the level-crossing detector or mechanical response of the switch.

We choose the following parameters:

$$C = 0.1 [\mu\text{F}], R = 40 [\text{k}\Omega], E = 3.0 [\text{V}], v_r = 2.7 [\text{V}], T = 2 [\text{ms}], \tau_d = 0.8 [\text{ms}] \quad (1)$$

Figure 3(a) shows the chaotic behavior of the system with these parameter values during the laboratory experiment. Since the chaos is ergodic, a form shown in Fig. 3(b) is visible in the corresponding return map. Since the mathematical model of this circuit can be rigorously derived [Oda et al., 2007], an unstable fixed point v^* embedded in this chaotic map is easily obtained by solving the equation $v_k = v_{k+1}$. In the following section, we attempt to stabilize the target v^* .

Experimental results: We choose E as a control parameter and design the linear controller proposed by [Oda et al., 2007]. We design the control gain such that the pole of the characteristic equation becomes equal to zero. For the parameter set in Eq. (1), we obtain a control input as a parameter perturbation $u_k = -3.9 \times (v_k - v^*)$.

Figure 4 shows a circuit diagram of the controller. The capacitor voltage $v(t)$ is applied to the sample/hold circuits at each clock pulse. The subtracter computes the difference between v_k and v^* . The inverting amplifier provides the control gain. The final block summing amplifier adds the control input u_k as a bias of E . The final output v_{out} is a rectangular wave with a fixed frequency $1/T$.

Since the controller is based on the linear dynamical property of the fixed point, a control limiter is used to suppress an unnecessarily large control input. We adopt a scheme in which u_k is fed back to E when the holding value v_{out} approaches the target sufficiently; otherwise, u_k is assumed to be zero. We realize this scheme by introducing a window comparator to monitor the difference between v_k and v^* . We set the threshold of this comparator as $|v_k - v^*| = 0.5$ [V].

The result of the laboratory experiment with this controller is shown in Fig. 5; here, (a) and (b) show the observed signal of v_k and the steady state of the mapped point on the return map, respectively. The transient behaviors of the control inputs v_k and u_k are shown in Fig. 5(c). After the transition, u_k converges to zero and v_k becomes a regular periodic wave, as shown in Fig. 5(a).

Conclusions: We have modeled a circuit realization of a chaotic system which includes the

delay time for switching action. From the above results, it is obvious that the controller responds rapidly and absorbs the switching delay efficiently.

References

- [Banerjee et al., 2004] Banerjee, S. Parui, S. & Gupta, A. [2004] “Dynamical Effects of Missed Switching in Current-Mode Controlled DC-DC Converters,” *IEEE Trans. Circuits and Systems*, **51**, 649–654.
- [Bernardo & Tse, 2002] Bernardo, M. di & Tse, C. K. [2002] “Chaos in Power Electronics: An overview,” *Chaos in Circuits and Systems*, World Scientific, Chap. 16.
- [Chen, 1999] Chen, G. [1999] “Controlling Chaos and Bifurcations in Engineering Systems,” ed. G Chen, CRC Press.
- [Oda et al., 2007] Oda, N. Kousaka, T. Ueta, T. & Ma, Y. [2007] “Controlling Chaos in a Simple Circuit with Switching Delay,” *Journal of Signal Processing*, **11**, 329–332.

Figures

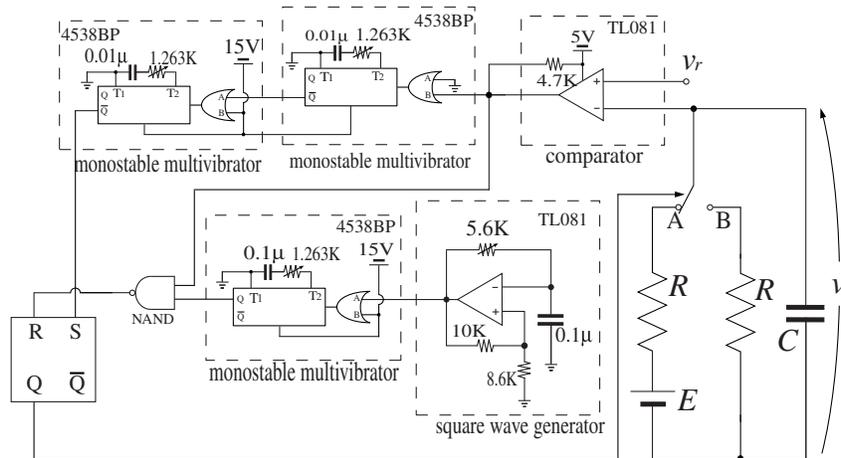


Figure 1: Circuit realization of chaotic switching system.

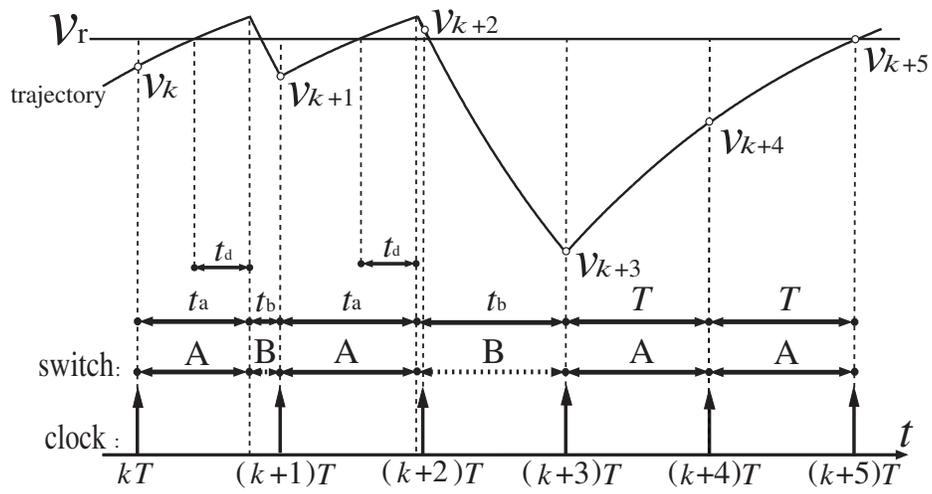


Figure 2: Behavior of the orbit with switching delay.

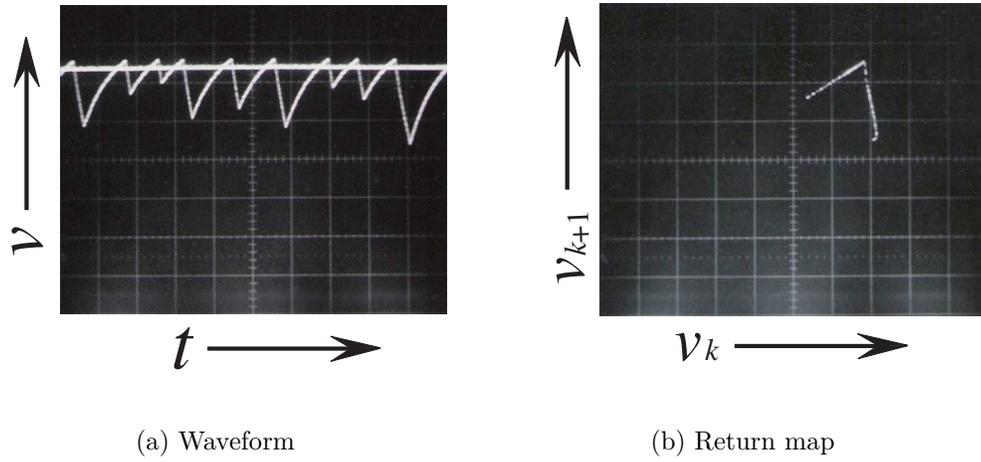


Figure 3: Chaotic attractor. ($t:2$ [ms/div], $v:0.5$ [V/div], $v_k:0.5$ [V/div], $v_{k+1}:0.5$ [V/div])

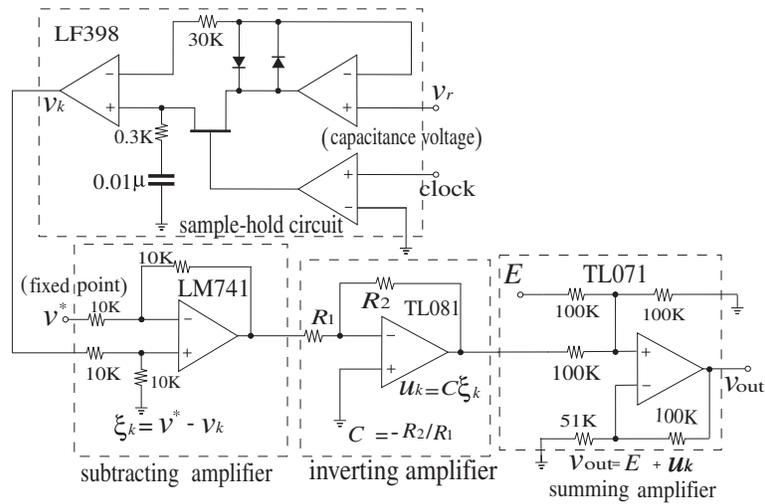
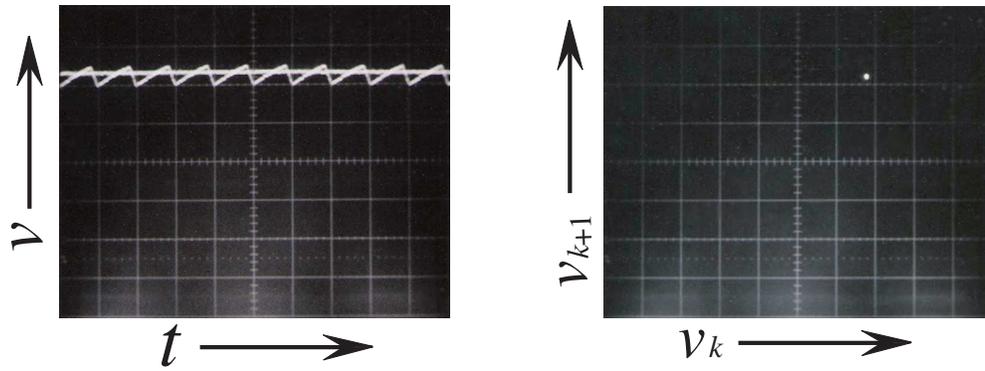
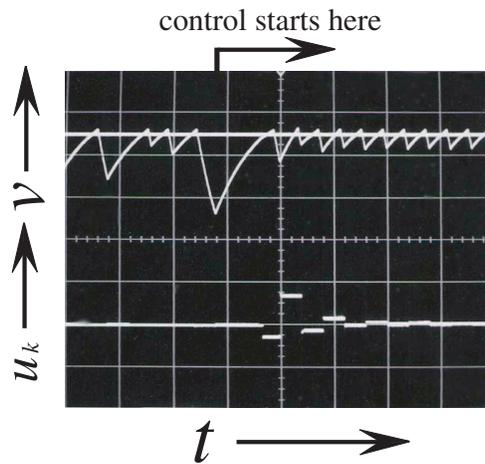


Figure 4: Block diagram of controller.



(a) Waveform

(b) Return map



(c) Transient behavior of control waveform and control input

Figure 5: Stabilized orbit.