Bifurcation analysis of forced hybrid dynamical systems
(强制外力を印加したハイブリッド力学系の分歧解析)

There are plenty of studies on nonlinear differential equations derived from mechanics, electronics, biology, and so on. These systems often exhibit a rich variety of responses, i.e., stable and unstable periodic orbits, quasi-periodic orbits, and chaos. Among them, bifurcations play a very important role, e.g., a tangent bifurcation generates or extinguishes a couple of a stable fixed point and a saddle point, a period-doubling bifurcation unstabilizes a stable fixed point and generates 2-periodic points, and a Neimark-Sacker bifurcation changes the stability of a fixed point with generating or extinguishing an invariant closed curve.

Recently, on the other hand, hybrid dynamical systems (HDSs) are actively studied. An HDS combines a continuous-time dynamical system and a discrete-time dynamical system; this combination is useful to construct the practical systems such as neural dynamics, conflict systems of objects, circuits including a diode and so on. However, we could find few studies analyzing forced HDSs, which are the HDSs containing one or more external forces in themselves.

In this thesis, we focus on forced hybrid dynamical systems (FHDSs), explore the method to solve the bifurcation problems of FHDSs, and finally show some examples of the results of bifurcation analyses.

In Chapter 2, we confirm the basic mathematical definitions of FHDSs, the Poincaré map of their trajectory, asymptotic stability, and so on. In Chapter 3, We extract the cause why we cannot apply the previous method to FHDSs, and investigate the way how to calculate the exact value of the Jacobian matrix of the Poincaré map. In Chapter 4, we see the results of the analysis for the practical FHDSs: a forced Izhikevich neuron model and its two-coupled model, a forced Alpazur oscillator, a Duffing circuit with an iron-cored inductor, and a single-junction superconducting quantum interferometer with a vibrating external flux. In Chapter 5, we consider the global bifurcation problem of 2-dim FHDSs. We finally see a result of the analysis for the simple FHDS. In Chapter 6, let us summarize our study and enumerate the problems for future work.