Optimally Selecting The Location Of A Multiple Of D-STATCOMs For The Improvement Of SARFIx Due To Faults In The IEEE 33-Bus Distribution System

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Abstract: The paper introduces a new method for optimizing the placement of a multiple of D-Statcoms for voltage sag mitigation in distribution systems. The D-Statcom’s placement is optimally selected not only for improving system voltage sag caused by a single fault event but also for all possible fault events in the system of interest. Therefore, D-Statcom’s placement is optimized in a problem of optimization where the objective function is to minimize the system voltage sag index – SARFIx. D-Statcom’s effectiveness for voltage sag mitigation is modeled basing on the method of Thevenin’s superimposition for the problem of short-circuit calculation in distribution systems. This paper considers the case of using a multiple of D-Statcoms with a proposed voltage compensating principle that can be practical for large-size distribution systems. The paper uses the IEEE 33-buses distribution feeder as the test system for voltage sag simulation and influential parameters to the outcomes of the problem of optimization are considered and discussed.

Keywords: Distribution System, Voltage Sag, SARFIx, Distribution Synchronous Compensation – D-Statcom

1. Introduction

According to IEEE 1159 [1], voltage sag/dip is a phenomenon of power quality (PQ) in which the rms value of the voltage magnitude drops below 0.9 p.u. in less than 1 minute. The main cause which is account of more than 90% voltage sag events is the short-circuit in the power systems. Solutions for voltage sag mitigation [2, 3] have generally been classified as two approaches [4] named “distributed improvement” and “central improvement” (or systematic improvement). The first is mainly considered for improving a single load while the latter is introduced for systematically improving PQ in the distribution system that is mainly interested by utilities. Either approaches have recently used custom power devices (CPD) [2] such as inverter-based voltage sources like the distribution static synchronous compensator (D-Statcom) as their cost has gradually decreased.

In reality, researches using D-Statcom for voltage sag mitigation have mainly been introduced for “distributed improvement” approach where dynamic modeling of D-Statcom is developed with main regard to D-Statcom’s controller design improvement [5-8] for mitigating PQ issues at a specific load site. The introduction of researches for “central improvement” that normally deal with the state description of a D-Statcom is popularly given as a current

with voltage quality in steady-state operation and loss reduction. [12] deals with the mitigation of various PQ issues including voltage sag using D-Statcom using multi-objective optimization approach, but such an optimization can rarely get the best performance for voltage sag mitigation only. [13] deals directly with voltage sag mitigation, but the modeling of D-Statcom for short-circuit calculation is still needed to improve. [14] introduced a good modeling of a CPD, but it is the case for dynamic voltage restorer (DVR) and the optimization of DVR application is just based on voltage sag event index.

This paper introduces a novel method for estimating the effectiveness of system voltage sag mitigation by the presence of a number of D-Statcoms in the short-circuit of a distribution system. This method optimizes the placement of D-Statcoms basing on minimizing a well-known system voltage sag index – SARFIx that consider all possible short-circuit events in a system of interest. In solving the problem of optimization, the modeling of a multiple of D-Statcoms simultaneously compensating system voltage sag in short-circuit events is introduced and discussed. The research uses the IEEE 33-buses distribution system as the test system. Short-circuit calculation for the test system as well as the modeling and solution of the problem of optimization are all programmed in Matlab.

For this purpose, the paper is structured as the following parts: The Section 2 introduces the modeling of D-Statcom for system voltage sag mitigation in the problem of short-circuit calculation in distribution system with its presence. The Section 3 introduces the problem of optimization. The results are analysed in the Section 4.

2. Modeling of D-Statcom with limited current for short-circuit calculation in distribution System

2.1 D-Statcom’s basic modeling for voltage sag mitigation

D-Statcom is a shunt connected FACTS device. The basic steady-state description of a D-Statcom is popularly given as a current

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source [3] injecting in a bus needed for voltage compensation. For mitigating voltage sag due to fault, the load voltage can be seen as the superposition of the system voltage and the voltage change due to the injected current by D-Statcom (Fig. 1).

![Fig. 1. Modeling D-Statcom for voltage sag mitigation](image)

For modeling the effectiveness of a multiple of D-Statcoms for system voltage sag mitigation, [14] introduced the application of the superposition principle according to the Thevenin theorem for the problem of short-circuit calculation in distribution system. It’s assumed that the initial state of the test system is the short-circuit without the presence of D-Statcoms. Thus, we have the system bus voltage can be calculated as follows:

\[ \Delta U_k = Z_{DS,k} \times I_{DS,k} + \sum_{j \in M, j \neq k} Z_{k,j} \times I_{DS,j} \]  

(10)

If the \( I_{DS,k} \) large enough, we assume the initial condition of voltage compensation is similar to [14] as follows:

\[ \Delta U_k = \hat{U}_k - \hat{U}_{sag,k} = 1 - \hat{U}_{sag,k} \]  

(11)

Replace (11) to (10) we have m equations to calculate m variables \( I_{DS,k} \) of m D-Statcoms. Solve this system of m equations, for the \( \hat{U}_1 \) initial condition of voltage compensation:

\[ \Delta U_1 = Z_{DS,1} \times I_{DS,1} + \sum_{j \in M, j \neq 1} Z_{1,j} \times I_{DS,j} \]  

(12)

where

\[ \hat{U}_1 = \Delta U_1 + \hat{U}_{sag,1} \]  

(13)

The typical V-I characteristic of a STATCOM is depicted in Fig. 2 showing that the STATCOM’s current can be within the range for a stable output voltage. If the STATCOM is connected to the location experiencing a deep sag, it can not boost the voltage up to 1p.u. for a given \( I_{DS,max} \). So, we assume that \( I_{DS} \) just takes \( I_{DS,max} \). As the result, the compensated voltage \( \Delta U_1 \) is

\[ \Delta U_1 = \hat{U}_{sag,k} - \hat{U}_L \]  

(3)

\[ |\Delta U_1| = I_{DS,max} \times Z_{th} = |\hat{U}_L - \hat{U}_{sag}| < |1 - \hat{U}_{sag}| \]  

(4)

2.2 Modeling of a multiple of D-Statcoms for system voltage sag mitigation

a. Generality

For modeling the effectiveness of a multiple of D-Statcoms for system voltage sag mitigation, [14] introduced the application of the superposition principle according to the Thevenin theorem for the problem of short-circuit calculation in distribution system. It’s assumed that the initial state of the test system is the short-circuit without the presence of D-Statcoms. Thus, we have the system bus voltage can be calculated as follows:

\[ [U^0] = [Z_{bus}] \times [I^0] \]  

(5); \n
\[ [\hat{U}^0] = \begin{bmatrix} \hat{U}_{sag,1} \\ \vdots \\ \hat{U}_{sag,n} \end{bmatrix} \]  

(6)

\[ [Z_{bus}]: \text{System bus impedance matrix calculated from the bus admittance matrix: } [Z_{bus}] = [Y_{bus}]^{-1}. \]  

If the short-circuit is assumed to have fault impedance, we can add the fault impedance to \( Z_{bus} \).

With the presence of D-Statcoms, according to Thevenin theorem, the bus voltage equation should be modified as follows [15]:

\[ \begin{bmatrix} \hat{U} \end{bmatrix} = [Z_{bus}] \times [I^0] + [\Delta U] = [Z_{bus}] \times [I^0] + [\Delta I] = [U^0] + [\Delta U] \]  

(7)

where

\[ \Delta U = [Z_{bus}] \times [\Delta I] \]  

(8) or

\[ \Delta U = [Z_{bus}] \times [\Delta I] = [Z_{bus}] \times [\Delta I] \]  

(9)

\( \Delta U_i \) Bus i voltage improvement \( (i=1,n) \) after adding the custom power devices in the system.

\( \Delta I_i \) Additional injected current to the bus i \( (i=1,n) \) after adding the custom power devices like D-Statcoms in the system.

b. Placing m D-Statcoms in the test system

Assume that \( M \) is the set of m buses to connect to D-Statcom (Fig. 3), so the column matrix of bus injected current \( \Delta I \) in (9) has m non-zero elements and n-m zero elements. From (9), for the

\[ \begin{bmatrix} \Delta U_1 \\ \vdots \\ \Delta U_n \end{bmatrix} = [Z_{bus}] \times [\Delta I] \]  

(14)

Replace (11) to (10) we have m equations to calculate m variables \( I_{DS,k} \) of m D-Statcoms. Solve this system of m equations, for the \( I_{DS,k} \) required values of \( I_{DS,k} \).
However, as above said, there’re definitely buses that need large \( I_{DS} \) to boost the bus voltage to 1 p.u. that is beyond D Statcom’s current limit. Therefore, for a given Statcom’s current limit \( I_{DSmax} \):
- If \( I_{DSk}^{\ast} \) is smaller than a given \( I_{DSmax} \), we use the value \( I_{DSk}^{\ast} \) to calculate the voltage upgrade of n-m buses without connecting to D Statcoms (\( I_{DSk} = I_{DSk}^{\ast} \)).
- If the given \( I_{DSmax} \) is smaller than \( I_{DSk} \), we use the given value \( I_{DSmax} \) as the current the D Statcom injects in bus \( k \) (\( I_{DSk} = I_{DSmax} \)) to calculate the voltage upgrade of n-m buses without connecting to D Statcoms and system voltage as (12).

\[
\Delta U_i = \sum_{k=1}^{n} z_{ik} \times I_{DSk}
\]

And finally, the system bus voltages after placing D Statcom are calculated as follows

\[
\bar{U}_i = \Delta U_i + U_{i_{\text{sag},i}}
\]

For better understanding about the above proposed modeling of the D Statcom’s voltage compensation in the short-circuit of distribution system, we consider the cases of using one or two D Statcoms as follows

b. Placing one D-Statcoms in the test system

Assuming a D-Statcom is placed at bus \( k \) (Fig. 4), the matrix of additional injected bus current in (9) only has one element at the row \( k \) that does not equal zero (\( \Delta U_k = I_{DS} \neq 0 \)). Other elements equal zero (\( \Delta U_i = 0 \) for \( i=1,n; i\neq k \)).

\[
\begin{bmatrix}
\Delta U_1 \\
\Delta U_2 \\
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
\end{bmatrix} \begin{bmatrix}
\Delta U_k \\
\Delta U_{\text{sag},k} \\
\end{bmatrix}
\]

\( \Delta U_i = \Delta U_i + U_{\text{sag},i} \) (13)

For better understanding about the above proposed modeling of the D Statcom’s voltage compensation in the short-circuit of distribution system, we consider the cases of using one or two D Statcoms as follows

For a given \( I_{DS} \), the injected \( I_{DS} \) injected to bus \( k \) is calculated by (9) as follows

\[
I_{DS} = I_{DSi} = \Delta U_i = \frac{\Delta U_i}{z_{ik}} = \frac{1}{z_{ik}} \times (1 - U_{\text{sag},i})
\]

If the given \( I_{DSmax} \) is lower than \( I_{DSi} \), the bus \( k \) voltage can increase only to a certain value \( U_{\text{sag},i} < 1 \text{ p.u.} \) as \( I_{DS} = I_{DSmax} \)

\[
U_{\text{sag},i} = \Delta U_i + U_{\text{sag},i} = I_{DSmax} \times z_{ik} + U_{\text{sag},i} < 1 \text{ p.u.}
\]

Other bus voltages \( (\bar{U}_i, i=1,n; i\neq k) \) can be calculated similar to (13) for one placing the D Statcom at bus \( k \) as follows

\[
\bar{U}_i = \Delta U_i + U_{\text{sag},i} = U_{\text{sag},i} + I_{DSmax} \times z_{ik} + U_{\text{sag},i}
\]

3. Problem Definition

3.1 IEEE 33-Bus Distribution System

For simplifying the introduction of the new method in the paper, the IEEE 33-bus distribution feeder (Fig. 6) is used as the test system because it just features a balanced three-phase distribution system, with three-phase loads and three-phase lines.

This research assumes base power to be 100MVA. Base voltage is 11kV. The system voltage is 1pu. System impedance is 0.1pu.

3.2 Short-circuit calculation

The paper only considers voltage sags caused by fault. Because the method introduced in this paper considers SARFI, we have to consider all possible fault positions in the test system. However, to simplify the introduction of the new method, we can consider only three-phase short-circuits. Other short-circuit types can be included similarly in the model if detailed calculation is needed.

Three-phase short-circuit calculations are performed in Matlab using the method of bus impedance matrix. The resulting bus voltage sags with and without the presence of D-Statcom can be calculated for different scenarios of influential parameters as analysed in Section 4
3.3 The problem of optimization

a. Objective function and constraints

In this research, D-Statcom’s effectiveness for total voltage sag mitigation is assessed basing on the problem of optimizing the location one or multiple D-Statcoms in the test system where the objective function is to minimize the System Average RMS Variation Frequency Index – SARFIx where X is a given rms voltage threshold [16].

\[
\text{SARFI}_x = \frac{\sum_{i=1}^{n} n_i x}{N} \Rightarrow \text{Min} \tag{21}
\]

where

- \(n_i x\): The number of voltage sags lower than \(X\%\) of the load \(i\) in the test system.
- \(N\): The number of loads in the system.

For a given fault performance (fault rate distribution) of a given system and a given threshold \(X\), SARFI\(_x\) calculation is described as the block-diagram in Fig. 7.

For this problem of optimization, the main variable is the scenario of positions (buses) where D-Statcoms are connected. We can see each main variable as a string of \(m\) bus numbers with D-Statcom connection out of the set of \(n\) buses of the test system. Therefore, the total scenarios of D-Statcom placement to be tested is the \(m\)-combination of set \(N\) (\(n=33\)):

\[
T_m = \frac{n!}{m!(n-m)!} = \frac{33!}{m!(33-m)!} \tag{22}
\]

For example, if we consider of placing 1 D-Statcom in the test system, we have \(m=1\) the main variable is \(k=1, 2…33\) and thus the total scenarios of D-Statcom position is

\[
T_1 = \frac{33!}{1!(33-1)!} = 33.
\]

If we consider the placement of 2 D-Statcom in the test system, we have \(m=2\) and the total scenarios for placing these two D-Statcoms is

\[
T_2 = \frac{33!}{2!(33-2)!} = 528.
\]

Each candidate scenario to be tested is a pair of buses number \(j\) and \(k\) out from 33 buses where the two D-Statcoms are connected (e.g. 1,2; 1,3;…).

The problem of optimization has no constraint, but an important parameter is be given is the limited current of D-Statcom. The modeling about how D-Statcom with a limited current compensates system voltage sag is introduced in Section 2.

b. Problem solving

For such a problem of optimization, with preset parameters (\(X\%\), number of D-Statcoms \(m\) and D-Statcom’s limited current), the objective function – SARFI\(_x\) is always determined. So, we use the method of direct search and testing all scenarios of D-Statcom positions \(T_m\). The block-diagram of solving this problem in Matlab is given in Fig.8.

![Fig. 7. Block diagram of SARFI\(_x\) calculation](image)

![Fig. 8. Block diagram of the problem of optimization](image)
Each scenario in $T_n$ is determined by counting a combination of m buses connected with D-Statcom out of n buses of the test system. For a candidate scenario k, we calculate the $I_{DS}$ of D-Statcom for verifying the D-Statcom’s limited current. The updated $I_{DS}$ is then used for calculate system voltage with the presence of D-Statcom and the resulting SARFI

In the block-diagram, input data that can be seen as the above said preset parameters. "postop" is the intermediate variable that fixes the scenario of D-Statcom position corresponding to the minimum SARFIx. The initial solution of objective function Min equals B (e.g. B=33) which is big value for starting the search process. All calculations are programmed in Matlab. The scenarios for parameters of fault events are considered.

4. Result Analysis

4.1 Preset parameters

The research considers the following preset parameters:
- For calculating $I_{DSmax}$, the fault performance which is fault rate distributed to all fault position. The paper uses uniform fault distribution as per [17] and fault rate = 1 time per unit period of time at fault position (each bus).
- For rms voltage threshold, the paper considers voltage sags so $X$ is given as 90, 80, 70, 50% of Un.
- For D-Statcom’s limited current, the paper considers $I_{DSmax} = 0.05, 0.1, 0.2$ p.u.

4.2 Placing one D-Statcom in the test system

The simplest case is that with one D-Statcom placed in the test system. Solving the problem of optimization considering above said preset parameters, step-by-step results are introduced. Such as we consider sag $X=80\%$, $I_{DSmax} = 0.1$ p.u. the optimal location of D-Statcom is bus 14. Sag frequency at all buses with or without D-Statcom optimally placed at bus 14 (min SARFI=80 = 12.0909) are plotted in Fig.9.

Consider other $X\%$ and $I_{DSmax}$, the results of SARFIx for all scenarios of D-Statcom placement are totally demonstrated in Fig.10 for different $X=50, 70, 80, 90\%$ at $I_{DSmax} = 0.1$ p.u and Fig. 11 for different $I_{DSmax}=0.05, 0.1, 0.2$ p.u. at $X=80\%$.

<table>
<thead>
<tr>
<th>$I_{DSmax}$ (pu)</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X=50%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>minSARFIx</td>
<td>9.9697</td>
<td>6.1212</td>
<td>5.1212</td>
<td>3.303</td>
</tr>
<tr>
<td>DS Bus</td>
<td>17</td>
<td>12</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>$X=70%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>minSARFIx</td>
<td>14.303</td>
<td>9.5758</td>
<td>7.4545</td>
<td>7.1818</td>
</tr>
<tr>
<td>DS Bus</td>
<td>12</td>
<td>13</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>$X=80%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>minSARFIx</td>
<td>16.4242</td>
<td>12.0909</td>
<td>9.4545</td>
<td>8.6364</td>
</tr>
<tr>
<td>DS Bus</td>
<td>12</td>
<td>14</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>$X=90%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>minSARFIx</td>
<td>20.7879</td>
<td>17.2727</td>
<td>12.4848</td>
<td>11.0609</td>
</tr>
<tr>
<td>DS Bus</td>
<td>13</td>
<td>10</td>
<td>10</td>
<td>8</td>
</tr>
</tbody>
</table>

4.3. Placing a multiple of D-Statcoms in the test system

The proposed method of modeling the system voltage sag mitigation for the case of using a multiple of D-Statcom in Section 2.2 can be illustrated for the case of using two D-Statcom. We know that the number of D-Statcoms should be suitable with the system size so that its voltage compensation is economically effective. For such a size of 33-bus test system, two D-Statcom can be used.

For the case of two D-Statcoms placed in the test system, solving the optimization problem, followings are step-by-step clarification and analysis of the results. We again start to consider the case with...
X=80% and \( I_{DS_{\text{max}}} = 0.1 \text{p.u.} \). The voltage sag frequency at all system buses are plotted for the case without D-Statcom and with two D-Statcoms in the Fig. 12. The two D-Statcoms are optimally located at bus 14 and bus 32 and the resulting minimum value of SARFI equals 8.7879. Fig. 13 also includes the voltage sag frequency for the case of using one D-Statcom as Fig. 9 for comparison.

In fact, the optimal placement of two D-Statcoms at buses 14 and 32 is searched from \( T_2 = 528 \) scenarios. The SARFI for \( X=80\% \) and \( I_{DS_{\text{max}}} = 0.1 \text{p.u.} \) is calculated for 528 scenarios as plotted in Fig. 13.

A scenario is a point with its ordinates equal to D-Statcom’s locations. Also, because we don’t consider the permutation for the pair of D-Statcom’s location (e.g. 1-2 is the same as 2-1), we only consider points on the triangle from the main diagonal of the matrix of scenarios of placement of 2 D-Statcoms. The points in the other triangle of the above said matrix are not considered and thus its objective function is given a high value (e.g. SARFI=33) for searching the minimum of SARFI. However, for better graphical description of SARFI as the function of two D-Statcoms placement, in the Fig. 14, the positions that are not considered are assigned the SARFI to equal zero.

Solving the problem of optimization for other preset parameters, the results are presented as the followings:

- Regarding the relation between SARFI\(X\) and the scenarios of 2 D-Statcom placement, Fig. 14 and 15 are presented to have a closer look on the influences of \( X\% \) to SARFI and \( I_{DS_{\text{max}}} \) to SARFI.

- Regarding the effectiveness on sag frequency of all system buses, the results by all preset parameters are described in Fig. 16 for \( X = 80\% \), \( I_{DS_{\text{max}}} = 0.05, 0.1, 0.2, 0.3 \text{p.u.} \) and Fig. 17 for \( X = 50, 70, 90\% \) and \( I_{DS_{\text{max}}} = 0.1 \text{p.u.} \).

Fig. 13, 14, 15 imply the optimal placement in the area of buses 10-15 and buses 25-32. In Fig. 16, D-Statcom’s stronger current results in smaller sag frequency. For \( I_{DS_{\text{max}}} = 0.2 \) and 0.3pu, the sag frequency is very small and for some buses it even equals zero. The sag frequency is very small in the area near the optimal scenario of D-Statcom location. For example, for \( I_{DS_{\text{max}}} = 0.3 \text{pu} \), optimal locations of D-Statcoms are bus 13 and bus 28 (see Table 2), and sag frequency is very small for buses 12-15 and 19-28. Fig. 17 shows an obvious influence of \( X \) as \( X \) is higher, the sag frequency is greater, but for \( X=50\% \), with two D-Statcoms, the sag frequency is very low (about 1.5). We know that for distribution system, the sag duration is defined mainly protection device tripping time and its typical time is 0.1s or greater. With regard to
the voltage ride-through curves [16]. X should be 50% or greater. For the size of distribution system like the 33-bus, using two D-Statcoms is good enough for mitigating almost voltage sags in the system. That’s why the paper takes the scenarios of two D-Statcom placement for modeling a multiple of D-Statcom mitigating system voltage sag for the 33-bus distribution system. Table 2 summarizes remarked results as follows:

Table 2. Results for using 2 D-Statcom

<table>
<thead>
<tr>
<th>SARFI (pu)</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS1 Bus</td>
<td>7.8485</td>
<td>2.6667</td>
<td>1.5758</td>
<td>1.5758</td>
</tr>
<tr>
<td>DS2 Bus</td>
<td>17</td>
<td>13</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>DS1 Bus</td>
<td>29</td>
<td>32</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>DS2 Bus</td>
<td>12.7273</td>
<td>5.1812</td>
<td>3.3939</td>
<td>3.0303</td>
</tr>
<tr>
<td>DS1 Bus</td>
<td>18</td>
<td>13</td>
<td>9</td>
<td>14</td>
</tr>
<tr>
<td>DS2 Bus</td>
<td>33</td>
<td>33</td>
<td>28</td>
<td>27</td>
</tr>
<tr>
<td>DS1 Bus</td>
<td>14</td>
<td>14</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>DS2 Bus</td>
<td>32</td>
<td>32</td>
<td>30</td>
<td>28</td>
</tr>
<tr>
<td>DS1 Bus</td>
<td>10</td>
<td>15</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>DS2 Bus</td>
<td>18</td>
<td>13</td>
<td>29</td>
<td>28</td>
</tr>
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For X=50, the SARFI does not improve for I

\[ X = 50\% \]

\[ X = 70\% \]

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