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Decoupling theorem in top productions/decays revisited

– To what extent can we understand it visually? –

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ABSTRACT

I revisit here the decoupling theorem in top-quark productions/decays, which states that the angular distribution of any final-particle produced in those processes does not depend on any possible nonstandard top-quark decay interactions at their leading order when certain conditions are satisfied. Towards a simple, intuitive and visual understanding of this theorem, I will study to what extent we could explain why such a theorem holds without relying on any specific/detailed calculations.

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1. Introduction

The top quark, the heaviest elementary particle of all those we have ever encountered, has a mass very close to the electroweak scale, which makes us expect that it plays an important role for understanding the spontaneous breakdown of this symmetry and works as a window for new more fundamental physics beyond the standard model. It will therefore be crucial to clarify the property of this quark in various different aspects. In order to carry out investigations for this purpose, we have to analyze its decay processes by examining the final products, since it turns into lighter particles right after being produced, even before the hadronization, due to its huge mass.

In performing such studies, we have discovered a remarkable fact that the angular distribution of the final charged-lepton ℓ^+ ($\ell = e, \mu$) in productions/decays of this quark depends only on possible nonstandard "production" interactions, in other words, it does not depend on any nonstandard "decay" interactions at their leading order [1]–[3], see also [4, 5]. This is what we call "the decoupling theorem in top productions/decays". This theorem is valuable in exploring new physics through analyzing possible anomalous top-quark couplings, since we are thereby able to study its production mechanism exclusively (i.e., without being affected by its decay interactions) via the ℓ^+ angular distribution. It is therefore meaningful to understand this theorem from more than one viewpoint.

We have not found any problem in our proof of this theorem [1]–[3], but we have to admit that we have not answered questions like "Can you explain it in an intuitive or visual way without using detailed calculations/formulas?", which we received many times after we published our papers. In order to compensate for this point, I will see in this article to what extent we can understand it without relying on any specific detailed calculations. Its original form is represented in terms of the initial-state momentum in top productions as the reference axis. This, however, makes visual arguments quite hard. Pointing out that we can study the theorem through polarized top-quark decays, I aim here to present some clear picture on how this theorem is born, which must be quite useful for other heavy-quark phenomenology and also instructive.

2. Basic framework and strategy

My strategy is to consider the theorem via decays of a polarized top quark, as mentioned in the first section. Generally, extracting the decay part from whole production/decay processes and treating that part independently is not justified, but it is possible in our case because the narrow-width approximation is expected to work well for the top-quark propagator. There, the top-quark spin direction is completely decided by the production process alone. Hence, if we can show that the angular distribution of the final particle around the top spin is not dependent on any nonstandard couplings responsible for the decay, we can in fact conclude that the angular distribution of the decay product of the top produced in the process is not affected by these nonstandard couplings, which means the theorem holds.

Let me show the base framework, which is the same as what we utilized in [1]–[3]: Once a top quark is produced, it decays immediately as $t \to bW^+$ in almost all cases. For describing those processes, I use the most general tbW coupling

$$
\Gamma^{\mu}_{tbW} = -\frac{g}{\sqrt{2}} \bar{u}_b(\boldsymbol{p}_b, s_b) \Big[\gamma^{\mu}(f_1^L P_L + f_1^R P_R)
$$
$$
- \frac{i\sigma^{\mu\nu}k_{\nu}}{M_W}(f_2^L P_L + f_2^R P_R) \Big] u_t(\boldsymbol{p}_t, s_t), \tag{1}
$$

where g is the $SU(2)$ coupling constant, k is the W-boson momentum, $P_{L/R} \equiv (1 \mp \gamma_5)/2$, $f_{1,2}^{L,R}$ are form factors (treated as constants) with $f_1^L = 1$ and $f_1^R = f_2^{L,R} = 0$ in the standard model (SM), while I assume that the W boson decays into a charged-lepton ℓ^+ and the corresponding neutrino ν_ℓ via the standard $V - A$ coupling. In the following, I set the z axis in the direction of the top-quark spin vector $\boldsymbol{s}_t$, and take it as the angular-momentum quantization axis. I neglect all the fermion masses except m_t, although the theorem still holds for $m_b \neq 0$ [3, 5].

Now, I express the momentum of the final particle $f\,(= b, \ell^+, \nu_\ell)$ as $\boldsymbol{p}_f$ and the unit vector of its direction as $\boldsymbol{n}_f$, i.e. $\boldsymbol{n}_f \equiv \boldsymbol{p}_f/|\boldsymbol{p}_f|$. Then, the angular distribution of f becomes a function of $\boldsymbol{s}_t$ and $\boldsymbol{n}_f$, denoted as $F(\boldsymbol{s}_t, \boldsymbol{n}_f)$, and this distribution must take the following form:[#1]

$$
d\Gamma/d\cos\theta_f = F(\boldsymbol{s}_t, \boldsymbol{n}_f) = C(1 + P\boldsymbol{s}_t\boldsymbol{n}_f) = C(1 + P\cos\theta_f), \tag{2}
$$

[#1] Note that $d\Gamma$ depends on $\boldsymbol{s}_t$ at most linearly because this $\boldsymbol{s}_t$ appears there through $u(\boldsymbol{p}_t, s_t)\bar{u}(\boldsymbol{p}_t, s_t) \propto 1 + \gamma_5 \boldsymbol{s}_t$.

where both C and P are constants[#2] depending generally on $f_{1,2}^{L,R}$, and θ_f is the angle between $\boldsymbol{s}_t$ and $\boldsymbol{n}_f$ as shown in Fig.1. Since there is no threshold coming from the momentum conservation in the processes we are considering, $\cos\theta_f$ can vary from -1 to $+1$. Therefore, the full width becomes $2C$, i.e., C must be positive, which leads to the following constraint on P

$$-1 \;\leq\; P \;\leq\; +1,$$

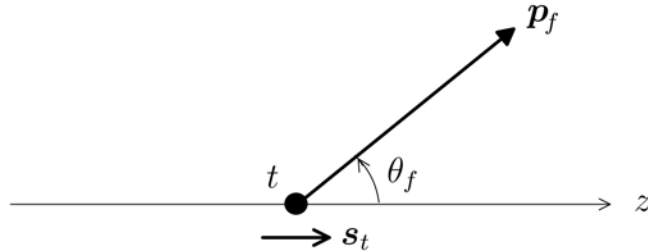as $d\Gamma/d\cos\theta_f$ must not be negative for any $\cos\theta_f$.

Figure 1: Basic quantities describing the angular distribution of $f\,(=b,\ell^+,\nu_\ell)$, whose momentum is $\boldsymbol{p}_f$. The z axis is set in the direction of the top-quark spin vector $\boldsymbol{s}_t$, and it is taken as the angular-momentum quantization axis. θ_f is the angle between $\boldsymbol{s}_t$ and $\boldsymbol{n}_f(\equiv \boldsymbol{p}_f/|\boldsymbol{p}_f|)$.

Here, if our final goal is $d\Gamma$ itself, we have to consider the anomalous-parameter dependences of both C and P. What interests us is however "the f distribution in whole top-production plus decay processes", where we need only $d\Gamma$ normalized by $\Gamma\,(=2C)$. We may therefore focus on P. There, if we can show that P is free from any anomalous tbW couplings, it does mean that the decoupling theorem holds, because they do not affect top-production processes as mentioned in the beginning. Generally, it will be totally difficult to do this via our simple arguments alone, but I find there is one possibility. That is to study if $P = \pm 1$ or not.

b-quark distribution

Let me go over the b-quark angular distribution in our framework as a clear example. Since my main concern here is in the leading nonstandard contributions

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[#2]P is indeed equivalent to the quantity known as "the spin analyzing power", but it is a mere unknown parameter in our simple discussions here.

coming from the SM-coupling plus those which can interfere with it, I assume that the emitted b is left-handed, to which f_1^L and f_2^R terms in eq.(1) contribute. As we understand from Fig.2, the b quark can be emitted both in the $+z$ direction ($\theta_b = 0$) and $-z$ direction ($\theta_b = \pi$), depending on whether W^+ is transverse or longitudinal. This means P is neither $+1$ nor -1. As mentioned, P depends generally on the parameters of the decay interaction, i.e., f_1^L and f_2^R in this case, which shows that the decoupling theorem may not hold for the b-quark distribution.[#3] This agrees with a conclusion of our preceding papers [1]–[3] that the final b-quark does not follow the theorem.
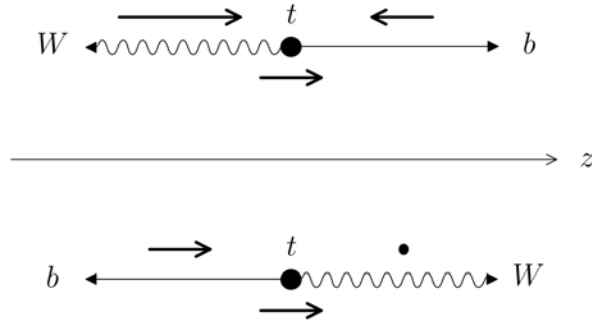

Figure 2: Allowed spin configurations for a left-handed b quark to be emitted in the $\pm z$ directions in the top-quark rest frame, where the thick arrows express the b and W spin vectors with nonvanishing z components and the blob above the lower wavy line expresses the W spin vector whose z component is zero.

3. Charged-lepton distribution

Let us proceed to the charged-lepton distribution. It might seem possible to study it the same way as in the previous section. It is indeed true that we can express the ℓ^+ angular distribution as

$$d\Gamma/d\cos\theta_\ell = C(1 + P\cos\theta_\ell) \tag{3}$$

through arguments like those leading to eq.(2). Then, if we thereby could show that $P = +1$ (or $P = -1$), it means that the decoupling theorem holds. In this

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[#3]Through the simple arguments here alone, we cannot avoid the possibility that P depends only on the SM coupling due to some reason. This is why I say "$\cdots$ theorem may not hold $\cdots$".

case, however, it is never easy to develop similar analyses since we have to treat a three-body final state. That is, when any of ℓ^+, b or ν_ℓ is emitted in a direction that is not parallel to the z axis (the spin quantization axis), the state of that particle becomes a superposition of $|s_z = +1/2\rangle$ and $|s_z = -1/2\rangle$ in quite contrast to the classical mechanics, and we will no longer be able to carry out clear discussions as for the b distribution.

Therefore I would like to add another assumption: the parent top quark emits W^+ and b parallel to its spin vector $\boldsymbol{s}_t$. Of course this does not always hold in the actual t-decay process, however it is never that unreasonable as an approximation in order to emphasize the characteristic feature of the process, since we can easily show according to the spin conservation that these two particles are most likely emitted along this axis. Figure 2 must be helpful in understanding it in the case where b is left-handed, and I will also discuss this point in the next section.
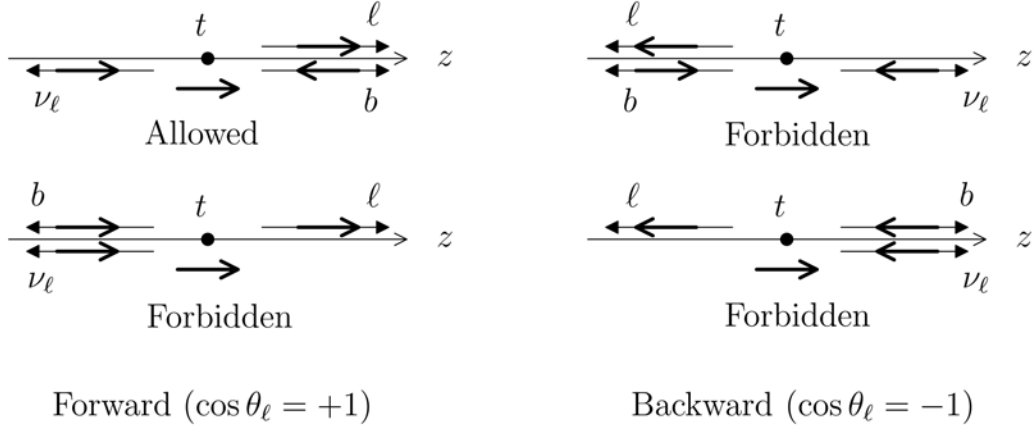

Forward $(\cos\theta_\ell = +1)$ Backward $(\cos\theta_\ell = -1)$

Figure 3: Allowed and forbidden spin configurations for the charged-lepton ℓ^+ moving along the z axis in the top-quark rest frame, where the thick arrows express the spin vectors. There might seem to exist other combinations in which ℓ and ν_ℓ move together in one direction, but such configurations are not realized through any W-boson boost in $W \to \ell\nu_\ell$.

Under this assumption, let us see whether ℓ^+ could move in the $+z/-z$ directions. Like the b distribution, I first assume that the emitted b is left-handed.

Since the final neutrino must also move parallel to the z axis in this case, no orbital angular momentum is involved. Then, according to the linear-momentum conservation, we can draw four final-particle configurations, and only one is allowed among them via the angular-momentum conservation as shown in Fig.3. This means that $\cos\theta_\ell = -1$ is not allowed and consequently P must be $+1$:

$$dΓ/d\cos\theta_\ell = C(1 + \cos\theta_\ell). \tag{4}$$

It is noteworthy that the decoupling theorem holds beyond the first order in the anomalous couplings in this situation.

Indeed the above result can be explicitly confirmed by the corresponding decay formula. That is, the four diagrams in Fig.3 are configurations in which the energy of ℓ^+ becomes the largest or the smallest, and the allowed one is the case where it is the lowest possible one (and the ν_ℓ energy is the largest). As is given in the appendix, the differential width is expressed as

$$\frac{dΓ}{d\varepsilon_\ell\, d\cos\theta_\ell} \sim f_+(\varepsilon_\ell)(1 + \cos\theta_\ell) + f_-(\varepsilon_\ell)(1 - \cos\theta_\ell) \tag{5}$$

apart from an overall coefficient, where $\varepsilon_\ell \equiv E_\ell/m_t$, and we have

$$f_+(\varepsilon_\ell) \propto |\, r_W f_1^L + f_2^R\,|^2 \quad \text{and} \quad f_-(\varepsilon_\ell) = 0 \tag{6}$$

for $\varepsilon_\ell = \varepsilon_\ell^{\min}(= r_W^2/2 \simeq 0.11)$, where $r_W \equiv M_W/m_t$, in complete agreement with eq.(4).

Similarly, if b is emitted via the f_1^R or f_2^L couplings and becomes right-handed, the upper left configuration in Fig.3 becomes "forbidden" and the lower left gets "allowed", which is the process where the ℓ^+ energy becomes the largest. This result again coincides with

$$f_+(\varepsilon_\ell) \propto |\, r_W f_1^R + f_2^L\,|^2 \quad \text{and} \quad f_-(\varepsilon_\ell) = 0 \tag{7}$$

for $\varepsilon_\ell = \varepsilon_\ell^{\max}(= 0.5)$.

It will be interesting to note that our arguments using Fig.3 still hold even if m_b is not neglected, since a nonvanishing m_b could reverse the b-quark spin in the right two graphs but those two remain to be "forbidden". Correspondingly, we can confirm

$$f_-(\varepsilon_\ell^{\max}) = f_-(\varepsilon_\ell^{\min}) = 0 \tag{8}$$

for eq.(14), where $\varepsilon_\ell^{\max,\min}$ are not the above ones but those for $m_b \neq 0$ (see the appendix). This is consistent with our proof that the decoupling theorem holds for $m_b \neq 0$ [3] (see also [5]). This kind of simple arguments based on the spin-conservation is often seen in explaining the form of the μ decay in the SM in the literature. Therefore, ours may seem a mere extension of them, showing that such an observation also works for the most general tbW couplings. Let us not forget, however, that understanding decay processes alone is not our final purpose here.

Before closing this section, I check how adequate our approximation on the W^+ and b directions which leads to the above arguments using the decay formula for $\varepsilon_\ell = \varepsilon_\ell^{\max}$ and $\varepsilon_\ell = \varepsilon_\ell^{\min}$ is. As shown, $f_-(\varepsilon_\ell)$ vanishes for $\varepsilon_\ell^{\max,\min}$ while is nonvanishing for a general ε_ℓ, see eqs.(15,16). Therefore, if $f_-(\varepsilon_\ell)$ increases sharply when ε_ℓ deviates from $\varepsilon_\ell^{\max}$ or $\varepsilon_\ell^{\min}$ even a little, it means our arguments have made full use of the very special property of $f_-(\varepsilon_\ell)$, and consequently they get
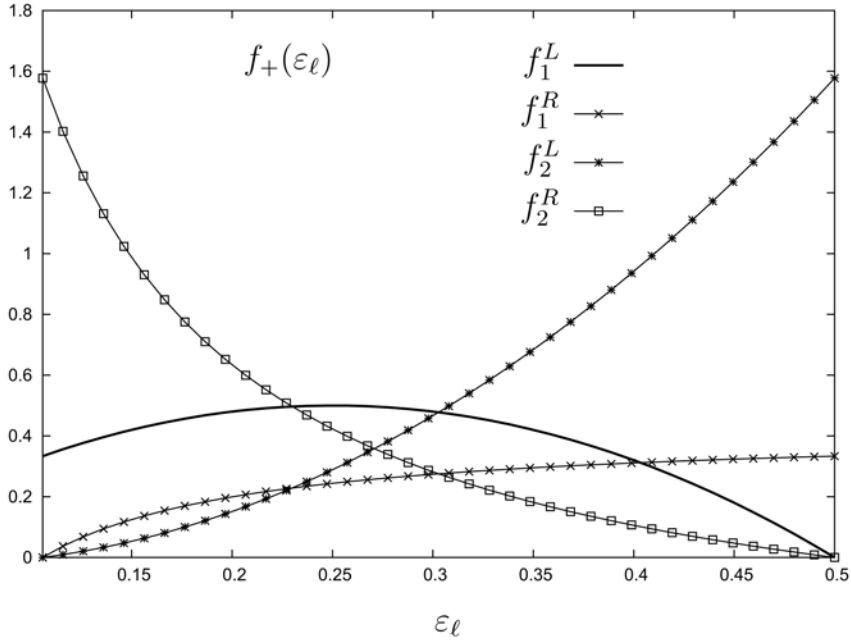

Figure 4: Four curves show the contribution of the $f_{1,2}^{L,R}$ terms to $f_+(\varepsilon_\ell)$. For example, the curve named f_1^L (solid curve) corresponds to $f_+(\varepsilon_\ell)$ in which f_1^L is set to be one and the others zero.
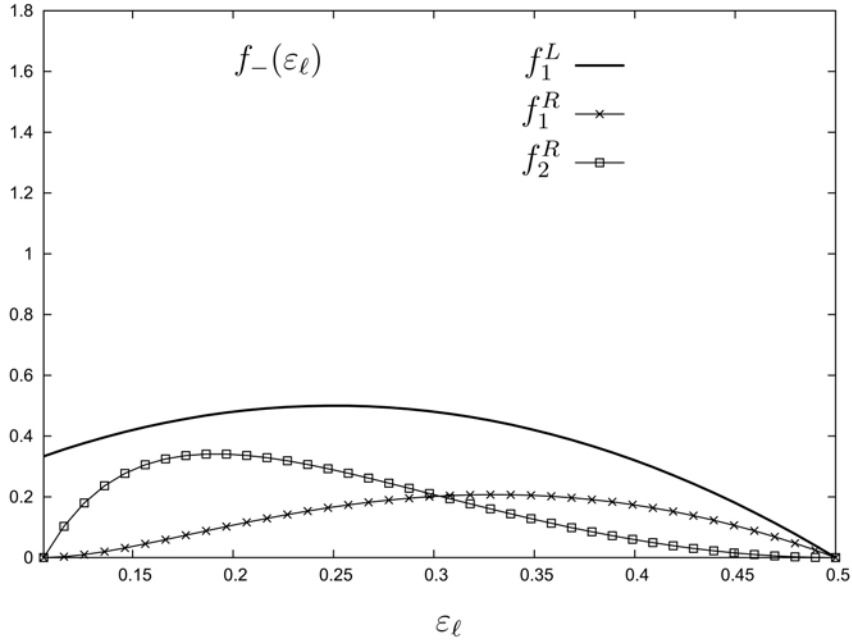
Figure 5: Two curves named $f_{1,2}^R$ show their contribution to $f_-(\varepsilon_\ell)$. The f_1^L curve in $f_+(\varepsilon_\ell)$ is also presented for comparison.

inappropriate for a general case. Fortunately, however, it does not seem to be the case as seen in Figs.4 and 5, where I have shown the ε_ℓ dependence of each term in $f_\pm(\varepsilon_\ell)$ using eqs.(15,16): Figure 4 is for f_+, and the curve named "f_1^L" there, e.g., corresponds to f_+ in which f_1^L is set to be one and the others zero. Similarly Fig.5 is for f_-, to which only $f_{1,2}^R$ contribute but I have added the f_1^L curve in f_+ for comparison. These two figures tell us that the $f_{1,2}^R$ terms in f_- remain small over the whole range of ε_ℓ.

4. More general cases

What can we know on the ℓ^+ distribution in a more general situation, in which the momenta of the final ℓ^+, ν_ℓ and b are not parallel to each other? As mentioned in the preceding section, any state of those particles is expressed as a superposition of its s_z eigenstates $|s_z = \pm 1/2\rangle$ unless its momentum is in the $+z$ or $-z$ direction. This quantum effect makes it totally difficult to understand the theorem visually, but this never means that we can find nothing there. In fact, we will be able to tell about a specific process as "favored/suppressed" instead of "allowed/forbidden".

First, let us remember the left-handed b-quark distribution considered in section 2. It will be then easy to understand that the theorem does not hold for the W angular distribution either since W is emitted in the direction opposite to b in the top-quark rest frame. However the transverse W with *helicity* $= -1$ (denoted hereafter as W_-) cannot be emitted in the $+z$ direction, i.e., $P = -1$ in eq.(2), and the longitudinal W (denoted as W_0) cannot be emitted in the $-z$ direction, i.e., $P = +1$:

$$d\Gamma/d\cos\theta_{W_-} = C_-(1 - \cos\theta_{W_-}), \qquad (9)$$

$$d\Gamma/d\cos\theta_{W_0} = C_0(1 + \cos\theta_{W_0}). \qquad (10)$$

This shows that the angular distributions of W_- and W_0 are both free from the anomalous decay-interaction couplings, i.e., they obey the decoupling theorem.

Now we know from the above equations that W_- is likely to be emitted backward while W_0 forward. Then we can study whether the final ℓ^+ is likely/unlikely to move in the $\pm z$ directions visually, noting that ℓ^+ is most likely to move in the W-boson spin direction in its rest frame. That is, ν_ℓ tends to move in the same direction as the W_- momentum and ℓ^+ in the opposite direction, while both of them from W_0 will move forward (but not parallel to each other) in the top rest frame. Figure 6 shows such configurations on the spins and momenta. Apparently, ℓ^+ is likely(unlikely) to be emitted forward(backward), indicating $P = +1$.
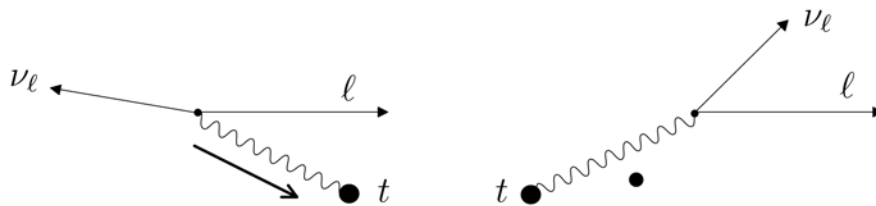

Figure 6: Favored spins and momenta configurations for a transverse W^+ (left side), a longitudinal W^+ (right side), and ℓ^+ in the top-quark rest frame. The b-quark line is not shown for simplicity.

We may say that those are analyses from a W^+-momentum viewpoint. We are also able to examine the issue from a viewpoint of the spin (z component)

conservation. I show the necessary spins/momenta configurations for ℓ^+ emissions in the $\pm z$ directions in Fig.7, where the thick solid arrows express the spin vectors in the eigenstates while those dotted-line arrows/circles mean that they are not in the eigenstates. It will not be hard there to understand that the left/right two configurations are favored/suppressed, taking account of the spin conservation.[#4] In this way, we find it quite plausible for ℓ^+ to move in the $+z$ direction but not in the $-z$ direction.

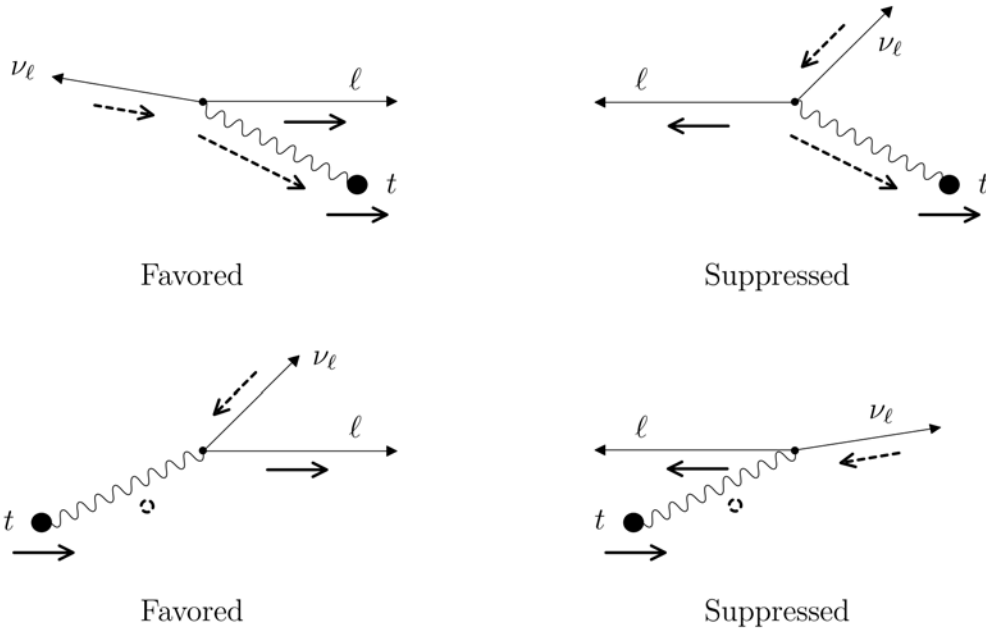| Favored	Suppressed

Favored	Suppressed

Figure 7: Spins and momenta configurations for the charged-lepton ℓ^+ emitted in the $+z$ direction (the left-side two diagrams) and $-z$ direction (the right-side two diagrams) in the top-quark rest frame, where the thick solid arrows express the spin vectors in the eigenstates, while those dotted-line arrows/circles mean that they are not in the eigenstates. The b-quark line is not shown for simplicity.

That is, Figures 6 and 7 both support P taking $+1$, which indicates that the decoupling theorem still holds in this more general case. This also seems to tell us that the same theorem is no longer valid for the ν_ℓ angular distribution since the lower left configuration is still valid while the upper right one turns "favored"

[#4]All the momenta in Fig.7 are on one common plane, so no orbital angular momentum (its z component) is involved again.

if we exchange ℓ and ν_ℓ in Fig.7. We have not studied the ν_ℓ distribution in our preceding papers, but this agrees with the results in [6]. Moreover, we can draw similar diagrams for right-handed b emissions. There, W_+ (W with *helicity* $= +1$) and W_0 are likely to be emitted forward and backward respectively, and neither ℓ^+ emission in the $-z$ direction (from W_0) nor that in the $+z$ direction (from W_+) is suppressed.

The above consideration can be applied to the standard-model term plus all the standard-nonstandard interference terms and show qualitatively how the ℓ^+ angular distribution gets to be proportional to $1 + \cos\theta_\ell$, but do not explain why the $|f_2^R|^2$ term does contribute both to f_+ and f_- as in eqs.(15,16) in spite of the fact that b is also left-handed in the f_2^R coupling. We need therefore some further analysis on this point.

First of all, note that the f_2^R term in eq.(1) can be divided into two different couplings, $\gamma^\mu(1 - \gamma_5)$ and $p_t^\mu(1 + \gamma_5)$, through some γ-matrix algebra and Dirac equations of t and b (using $m_b = 0$) [7]. The former structure is identical to f_1^L term, while the latter gives the following amplitude with the standard-model $\nu\ell W$ coupling:

$$\mathcal{M} \sim \bar{u}_b(\boldsymbol{p}_b, s_b)(1 + \gamma_5)u_t(\boldsymbol{p}_t, s_t) \cdot \bar{u}_\nu(\boldsymbol{p}_\nu, s_\nu)\slashed{p}_t(1 - \gamma_5)v_\ell(\boldsymbol{p}_\ell, s_\ell) \qquad (11)$$

Although the lepton part includes $\slashed{p}_t$, this is a kind of amplitude born through a scalar-boson (denoted as S) production/decay, i.e., ℓ^+ in this case is a product of a leptonic S decay. Therefore, the final charged lepton is always produced via a polarized-vector (W) decay in the f_1^L coupling, while the one in the f_2^R coupling receives contributions both from a polarized-vector-boson decay and a scalar-boson decay. Since the latter is not constrained by the same spin-polarization condition as f_1^L, then the pure f_2^R term could survive in f_- through the "scalar-exchange" amplitude. That is, the $|f_2^R|^2$ terms would break the theorem.

5. Summary

The decoupling theorem in top-quark productions/decays is a valuable tool in studying the property of this heavy quark in various aspects [1]–[3], [4, 5]. More

specifically, this theorem is quite helpful in exploring new physics beyond the standard model through analyzing possible anomalous top-quark couplings, since we are thereby able to look into its production mechanism without being affected by its decay interactions via the ℓ^+ angular distribution. It is therefore quite meaningful to grasp this theorem in many different ways, clarifying why such a theorem could exist in simple and visual manners.

Focusing on semileptonic decays of a polarized top quark $t \to bW^+ \to b\ell^+\nu_\ell$ instead of considering full top production and decay processes, I have here tried to explain this theorem without relying on any detailed calculations. I have shown first that the theorem can be understood precisely as a result of the spin z component conservation, once we assume that W^+ and b are emitted parallel to the top-quark spin vector in $t \to bW^+$ as a reasonable approximation in order to emphasize the characteristic feature of the process. Although I have treated the b quark as a massless particle through the main text for simplicity, those arguments are correct even when we do not neglect its mass.

I then studied more general cases, in which the final three fermions momenta are not parallel to each other. It is no longer possible to perform strict arguments in those cases, but still I found that the decoupling theorem is quite plausible from a W^+-momentum viewpoint and also from a spin-conservation viewpoint. There has been seen no contradiction between the present qualitative results and previous quantitative ones. I hope some visual picture on this theorem presented here is instructive and also useful for other heavy-quark phenomenology.

Acknowledgments

Appendix

I first present the angular and energy distribution of the final charged-lepton (ℓ^+) coming from $t \to bW^+ \to b\ell^+ \nu_\ell$ in the top-quark rest frame based on the interaction given in eq.(1) for the tbW coupling while the standard-model $V - A$ interaction for the $\nu\ell W$ coupling, keeping the b-quark mass. Adopting the narrow-width approximation for the W-boson propagator, the distribution is given as

$$\frac{d\Gamma}{d\varepsilon_\ell \, d\cos\theta_\ell} = \frac{g^4 m_t^2}{(32\pi)^2 r_W \Gamma_W} \Big[\, f_+(\varepsilon_\ell)(1 + \cos\theta_\ell) + f_-(\varepsilon_\ell)(1 - \cos\theta_\ell) \,\Big], \qquad (12)$$

where

$$\begin{aligned}
f_+(\varepsilon_\ell) =\ & 4\,|f_1^L|^2 \varepsilon_\ell (1 - r_b^2 - 2\varepsilon_\ell) + |f_1^R|^2 r_W^2 (2 - r_W^2/\varepsilon_\ell) \\
& - 4\,|f_2^L|^2 \varepsilon_\ell (r_W^2 - 2\varepsilon_\ell) - |f_2^R|^2 r_W^2 \big[\, 2 - (1 - r_b^2)/\varepsilon_\ell \,\big] \\
& - 4\,\mathrm{Re}(f_1^L f_1^{R*}) r_b r_W^2 - 8\,\mathrm{Re}(f_1^L f_2^{L*}) r_b r_W \varepsilon_\ell \\
& + 4\,\mathrm{Re}(f_1^L f_2^{R*}) r_W (1 - r_b^2 - 2\varepsilon_\ell) - 4\,\mathrm{Re}(f_1^R f_2^{L*}) r_W (r_W^2 - 2\varepsilon_\ell) \\
& - 2\,\mathrm{Re}(f_1^R f_2^{R*}) r_b r_W^3 / \varepsilon_\ell - 4\,\mathrm{Re}(f_2^L f_2^{R*}) r_b r_W^2, \qquad (13) \\
f_-(\varepsilon_\ell) =\ & |f_1^R|^2 \big[\, -2 r_W^2 (2 - r_b^2 + r_W^2) + 4(1 - r_b^2 + 2r_W^2)\varepsilon_\ell - 8\varepsilon_\ell^2 + r_W^4/\varepsilon_\ell \,\big] \\
& + |f_2^R|^2 \big[\, 2 - 2(2 - r_b^2)(r_b^2 - r_W^2) - 4(2 - 2r_b^2 + r_W^2)\varepsilon_\ell + 8\varepsilon_\ell^2 \\
& \quad - r_W^2 (1 - r_b^2)/\varepsilon_\ell \,\big] \\
& + 2\,\mathrm{Re}(f_1^R f_2^{R*}) r_b r_W \big[\, -2(1 - r_b^2 + r_W^2) + 4\varepsilon_\ell + r_W^2/\varepsilon_\ell \,\big], \qquad (14)
\end{aligned}$$

θ_ℓ being the angle between the ℓ^+ momentum and the top-quark spin, Γ_W is the W-boson total decay width, $r_b \equiv m_b/m_t$, $r_W \equiv M_W/m_t$, $\varepsilon_\ell \equiv E_\ell/m_t$ with E_ℓ being the ℓ^+ energy, and this "normalized" energy is restricted as

$$(\varepsilon_W - \kappa_W)/2 \le \varepsilon_\ell \le (\varepsilon_W + \kappa_W)/2$$

with $\varepsilon_W = (1 - r_b^2 + r_W^2)/2$ and $\kappa_W = \sqrt{\varepsilon_W^2 - r_W^2}$ [8] (see also [7]). I have compared $d\Gamma/d\cos\theta_\ell$ obtained by integrating eq.(12) on ε_ℓ with the corresponding formula in [6], and confirmed that there is no discrepancy between them.

These equations show that all the leading contributions, i.e. those including f_1^L, are only in $f_+(\varepsilon_\ell)$ even when we keep m_b finite. That is, the angular distribution of the final charged lepton around the top-quark spin is always proportional to $1 + \cos\theta_\ell$ in the top-quark rest frame at the leading order whatever form the tbW

coupling takes. This is equivalent to the decoupling theorem we found in our previous papers [1]–[3] since any top-decay interactions cannot affect the top-spin direction determined in production processes.

Let me next give the decay formula for a vanishing b-quark mass, which is directly related to our arguments in the main text. Under this approximation, the forms of $f_\pm(\varepsilon_\ell)$ become simpler as

$$
\begin{aligned}
f_+(\varepsilon_\ell) = {} & 4\,|f_1^L|^2\varepsilon_\ell(1-2\varepsilon_\ell) + |f_1^R|^2 r_W^2(2-r_W^2/\varepsilon_\ell) \\
& - 4\,|f_2^L|^2\varepsilon_\ell(r_W^2-2\varepsilon_\ell) - |f_2^R|^2 r_W^2(2-1/\varepsilon_\ell) \\
& + 4\,\mathrm{Re}(f_1^L f_2^{R*})r_W(1-2\varepsilon_\ell) - 4\,\mathrm{Re}(f_1^R f_2^{L*})r_W(r_W^2-2\varepsilon_\ell), \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
f_-(\varepsilon_\ell) = {} & |f_1^R|^2[\,-2r_W^2(2+r_W^2) + 4(1+2r_W^2)\varepsilon_\ell - 8\varepsilon_\ell^2 + r_W^4/\varepsilon_\ell\,] \\
& + |f_2^R|^2[\,2 + 4r_W^2 - 4(2+r_W^2)\varepsilon_\ell + 8\varepsilon_\ell^2 - r_W^2/\varepsilon_\ell\,], \qquad (16)
\end{aligned}
$$

and we have $\varepsilon_W = (1+r_W^2)/2$, $\kappa_W = (1-r_W^2)/2$, leading to $\varepsilon_\ell^{\max} = 1/2$ and $\varepsilon_\ell^{\min} = r_W^2/2$ with

$$
f_+(\varepsilon_\ell^{\max}) = 2(1-r_W^2)|\,r_W f_1^R + f_2^L\,|^2, \qquad f_-(\varepsilon_\ell^{\max}) = 0, \qquad (17)
$$

$$
f_+(\varepsilon_\ell^{\min}) = 2(1-r_W^2)|\,r_W f_1^L + f_2^R\,|^2, \qquad f_-(\varepsilon_\ell^{\min}) = 0, \qquad (18)
$$

which have been used in section 3.

Finally, there should be one comment on the narrow-width approximation for the W-boson propagator, which we have adopted through our work as mentioned in the beginning. This approximation is indeed quite helpful in calculations of the ℓ^+ distribution, but the resultant structure that all the leading terms in the anomalous couplings are proportional to $1 + \cos\theta_\ell$ is unchanged even if we do not use it, as studied in [5, 7].

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