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# 論 文 内 容 要 旨

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| 学位論文題目 | Numerical Approaches for Nonlinear Singular Systems<br>—Hidden Dynamics and Slow–Fast Dynamics—<br>(特異な非線形力学系の数値解析 ～隠された力学系と遅速力学系～) | | |


内容要旨

Nonlinear dynamical systems appear transversally across various fields such as physics, electrical and electronics, and biology, where systems are described as ordinary differential equations or difference equations. The time evolution solutions of dynamical systems change depending on the parameters of the system, often generating equilibrium points and periodic solutions. These states possess stability and can be classified as stable or unstable. Discussing the stability of states in a system helps determine which stable state a given initial condition will converge to. When system parameters change, certain equilibrium states may alter their stability, a phenomenon known as bifurcation. Identifying parameters where bifurcation occurs is crucial for understanding the global behavior of the system. Calculating the set of parameters where bifurcation occurs allows for the avoidance or induction of specific responses.

Research in dynamical systems often require numerical computations, as solutions are not always analytically obtainable. However, there exist systems for which classical numerical methods of dynamics are inapplicable. In hidden dynamical systems, the system appears to have a normal equilibrium state. However, there may be attractors that are not observed unless initial values are given in a very small range or far from the attractor, referred to as hidden dynamics. In slow-fast dynamical systems, the singular perturbation characteristics of the system may render standard numerical methods for dynamical systems completely unusable. Also, these systems may show periodic response behaviors, known as canards, which are not possible in normal dynamical systems. Due to their singular perturbation characteristics, it is difficult to apply traditional numerical methods of dynamical systems.

This thesis aims to explore effective numerical solutions for hidden and slow-fast dynamics, clarifying the response and bifurcation structure of specific systems. Chapter 2 reviews the mathematical definition of ordinary dynamical systems and explains basic numerical and bifurcation computation methods. In Chapter 3, we discuss efficient implementation methods for the bifurcation computation methods shown in Chapter 2 using Python and C++ languages. Chapter 4 discusses the bifurcation structure and hidden dynamics of the generalized Hénon map using the numerical methods shown in Chapters 2 and 3. This chapter uses only the classical computational methods of dynamical systems explained initially to describe hidden dynamics. Subsequently, Chapter 5 discusses the multivibrator, a type of slow-fast dynamical system. The numerical methods shown in Chapters 2 and 3 are difficult to apply to slow-fast dynamical systems. Here, we also explain numerical methods for calculating canards and their occurrence parameters, demonstrating the specific bifurcation structure and the existence of canards. Finally, Chapter 6 summarizes our research, remaining issues, and future works.